انتگرال‌های نامعین زیر را حل کنید.

الف) $\int x^2 \sqrt[5]{1-2x^3}\, dx = -\frac{1}{6}\int \overbrace{-6x^2 \underbrace{\sqrt[5]{1-2x^3}}_{u}\, dx}^{du} = -\frac{1}{6}\int \sqrt[5]{u}\, du = -\frac{1}{6}\int u^{\frac{1}{5}}\, du$

$1-2x^3 = u \rightsquigarrow -6x^2\, dx = du$

$= -\frac{1}{6}\left(\frac{u^{\frac{1}{5}+1}}{\frac{1}{5}+1}\right) + C = -\frac{\frac{1}{6}}{\frac{6}{5}}\, u^{\frac{6}{5}} + C$

$\rightarrow \frac{-5}{36}\sqrt[5]{(1-2x^3)^6} + C$

ب) $\int \frac{x+1}{\sqrt[3]{\underbrace{x^2+2x+3}_{u}}}\, dx = \frac{1}{2}\int \frac{\overbrace{2(x+1)\, dx}^{du}}{\sqrt[3]{\underbrace{x^2+2x+3}_{u}}} = \frac{1}{2}\int \frac{du}{\sqrt[3]{u}} = \frac{1}{2}\int u^{-\frac{1}{3}}\, du$

$= \frac{1}{2}\left(\frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}\right) + C = \frac{1}{2}\left(\frac{u^{\frac{2}{3}}}{\frac{2}{3}}\right) + C = \frac{3}{4}\sqrt[3]{u^2} + C = \frac{3}{4}\sqrt[3]{(x^2+2x+3)^2} + C$

$x^2 + 2x + 3 = u \rightsquigarrow (2x+2)\, dx = du \Rightarrow 2(x+1)\, dx = du$

ب) $\int \frac{\sin \overset{u}{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int \frac{\sin \overset{u}{\sqrt{x}}}{2\sqrt{x}}\,\overset{du}{dx} = 2\int \sin u\,du = -2\cos u + C$

$= -2\cos\sqrt{x} + C$

$\sqrt{x} = u \rightsquigarrow \frac{1}{2\sqrt{x}}\,dx = du$

ت) $\int \cos x \sqrt{\underbrace{2-\sin x}_{u}}\,dx = -\int -\cos x \sqrt{\underbrace{2-\sin x}_{u}}\,dx = -\int \sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du$

$= -\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{2}{3}\sqrt[2]{u^3} = -\frac{2}{3}\sqrt[2]{(2-\sin x)^3} + C$ $\quad du$

$2-\sin x = u \rightsquigarrow -\cos x\,dx = du$

ث) $\int \frac{\sin x}{۲+\cos x}\,dx = -\int \frac{\overbrace{-\sin x\,dx}^{du}}{\underbrace{۲+\cos x}_{u}} = -\int \frac{du}{u} = -\ln|u| + c = -\ln|۲+\cos x| + c$

$۲+\cos x = u \longrightarrow -\sin x\,dx = du$

ج) $\int \underbrace{\cos x\, e^{\overbrace{\sin x}^{u}}\,dx}_{du} = \int e^{u}\,du = e^{u} + c = e^{\sin x} + c$

$\sin x = u \longrightarrow \cos x\,dx = du$

ج) $\displaystyle\int \frac{\text{۴}\,dx}{e^{x}+e^{-x}} = \int \frac{\text{۴}\,dx}{e^{x}+\frac{1}{e^{x}}} = \int \frac{\text{۴}\,dx}{\frac{e^{\text{۲}x}+1}{e^{x}}} = \int \frac{\text{۴}e^{x}dx}{(e^{x})^{\text{۲}}+1} = \text{۴}\int \frac{\overbrace{e^{x}dx}^{du}}{(\underbrace{e^{x}}_{u})^{\text{۲}}+1}$

$\displaystyle = \text{۴}\int \frac{du}{u^{\text{۲}}+1} = \text{۴}\,\mathrm{Arctan}\,u + C = \text{۴}\,\mathrm{Arctan}(e^{x}) + C$

$e^{x} = u \longrightarrow e^{x}dx = du$

ح) $\displaystyle\int \frac{\text{۲}e^{x}}{\underbrace{1+e^{x}}_{u}}dx = \text{۲}\int \frac{\overbrace{e^{x}dx}^{du}}{\underbrace{1+e^{x}}_{u}} = \text{۲}\int \frac{du}{u} = \text{۲}\ln|u| + C = \text{۲}\ln|1+e^{x}| + C$

$1+e^{x} = u \longrightarrow e^{x}dx = du$

خ) $\displaystyle\int (x+1)\, 2^{\overset{u}{x^2+2x-1}}\, dx = \frac{1}{2}\int \underbrace{2(x+1)\, 2^{\overset{u}{x^2+2x-1}}\, dx}_{du} = \frac{1}{2}\int 2^u\, du$

$\displaystyle = \frac{1}{2}\left(\frac{2^u}{\ln 2}\right) + C = \frac{1}{2}\left(\frac{2^{x^2+2x-1}}{\ln 2}\right) + C$

$x^2 + 2x - 1 = u \rightsquigarrow (2x+2)\, dx = du \rightsquigarrow 2(x+1)\, dx = du$

د) $\displaystyle\int \frac{\sin(\overset{u}{\ln x})}{x}\, dx = \int \underbrace{\frac{\sin(\overset{u}{\ln x})\, dx}{x}}_{du}$ $\qquad x>0$

$\displaystyle = \int \sin u\, du = -\cos u + C = -\cos(\ln x) + C$

$\ln x = u \rightsquigarrow \frac{1}{x}\, dx = du$

ز) $\int \frac{x^۲ dx}{\sqrt{x^۳+1}} = \frac{1}{۳}\int \frac{\overbrace{۳x^۲ dx}^{du}}{\sqrt{\underbrace{x^۳+1}_{u}}} = \frac{1}{۳}\int \frac{du}{\sqrt{u}} = \frac{1}{۳}\int u^{-\frac{1}{۲}} du$

$= \frac{1}{۳} \frac{u^{\frac{1}{۲}}}{\frac{1}{۲}} + C = \frac{۲}{۳}\sqrt{u} + C$

$= \frac{۲}{۳}\sqrt{x^۳+1} + C$

$x^۳ + 1 = u \rightarrow ۳x^۲ dx = du$

ر) $\int \frac{dx}{\sqrt{۹-۴x^۲}} = \int \frac{dx}{\sqrt{۳^۲ - (\underbrace{۲x}_{u})^۲}} = \frac{1}{۲}\int \frac{\overbrace{۲\, dx}^{du}}{\sqrt{۳^۲-(\underbrace{۲x}_{u})^۲}} = \frac{1}{۲}\int \frac{du}{\sqrt{۳^۲-u^۲}}$

$= \frac{1}{۲}\left(\text{ArcSin}\,\frac{u}{۳}\right) + C = \frac{1}{۲}\left(\text{ArcSin}\,\frac{۲x}{۳}\right) + C = \frac{1}{۲}\text{ArcSin}\,\frac{۲x}{۳} + C$

$۲x = u \rightarrow ۲\,dx = du$

ii) $\int \frac{dx}{\sqrt{6x - x^2}} = \int \frac{dx}{\sqrt{-(x^2-6x)}} = \int \frac{dx}{\sqrt{-(x^2-6x+9-9)}} = \int \frac{dx}{\sqrt{-(x^2-6x+9)+9}}$

$= \int \frac{dx}{\sqrt{9-(x^2-6x+9)}} = \int \frac{dx}{\sqrt{9-(x-3)^2}} = \int \frac{\overset{du}{dx}}{\sqrt{3^2-(\underset{u}{x-3})^2}}$

$= \int \frac{du}{\sqrt{3^2-u^2}} = \text{ArcSin}\,\frac{u}{3} + C = \text{ArcSin}\,\left(\frac{x-3}{3}\right) + C$

$x - 3 = u \longrightarrow dx = du$

ز) $\int \frac{\overbrace{dx}^{du}}{x(\underbrace{1+\ln x}_{u})} = \int \frac{du}{u} = \ln|u| + C = \ln|(1+\ln x)| + C$

$1 + \ln x = u \rightarrow \frac{1}{x}\,dx = du$

س) $\int \frac{\overbrace{(۲x+۳)\,dx}^{du}}{\underbrace{x^۲+۳x+۴}_{u}} = \int \frac{du}{u} = \ln|u| + C = \ln|x^۲+۳x+۴| + C$

$x^۲ + ۳x + ۴ = u \rightarrow (۲x+۳)\,dx = du$